

## Fine-tuning in GGM and the 126 GeV Higgs particle

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**ABSTRACT:** In this paper we reanalyze the issue of fine-tuning in supersymmetric models which feature Generalized Gauge Mediation (GGM) in the light of recent measurement of the mass of the light Higgs particle and taking into account available data on the value of the muon magnetic moment  $g_\mu - 2$ . We consider GGM models with 3, 5 and 6 input parameters and reduce the fine-tuning by assuming simple relations between them at the high scale. We are able to find solutions which give the correct value of the light Higgs mass and are less fine-tuned than models with standard gauge mediation (and with gravity mediation), however one never finds fine-tuning measure lower than about  $10^2$  if one neglects the data on  $g_\mu - 2$  and about four times more if one takes the constraint given by  $g_\mu - 2$  into account. In general the current  $g_\mu - 2$  data push the models towards the high fine-tuning region. It is interesting to note, that once one removes the contributions to the finetuning induced by  $\mu$  and  $B_\mu$ , then in the case with neglected  $g_\mu - 2$  constraint one can easily find realistic vacua with fine-tuning of order 1 or lower, while the fine-tuning remains always large when the  $g_\mu - 2$  constraint is enforced. One should note, that in the last case even a small shift of the light Higgs mass towards smaller values both reduces fine-tuning and helps to improve agreement of a model with  $g_\mu - 2$  data.

**KEYWORDS:** Supersymmetry Phenomenology

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## 1 Introduction

The discovery of the Higgs boson at LHC with mass of about 126GeV seems to favour MSSM which predicts that the lightest Higgs boson can't be much heavier than Z boson. However Higgs mass this far from Z mass requires large radiative corrections which have to come from heavy supersymmetric particles. Such heavy sparticles reintroduce some fine-tuning in MSSM [1, 2] because large supersymmetric parameters also have to cancel out to secure electroweak symmetry breaking at the correct energy scale, thus threatening the motivation of SUSY as solution to naturalness problem.

In MSSM large fine-tuning originates from requiring sparticles heavy enough to generate observed Higgs mass. In mSUGRA models the simplest way of increasing Higgs mass is to get maximal stop mixing which increases dominant stop correction, but requires large negative A-terms. In gauge mediated models [3–13] however, usually only negligible A-terms are generated at the SUSY breaking scale. So the Higgs mass can be increased only using non-universality of scalars and fermions through subdominant corrections.

General gauge mediation [14] has already been studied in terms of phenomenology [15, 16] and specifically fine-tuning [17, 18]. However, we shall reanalyze the issue of fine-tuning taking into account the recent measurement of Higgs boson mass and data on the anomalous magnetic moment of the muon. In this work we use new realization (see the

appendix) of a well known algorithm used to find SUSY spectra [19–21], to check how much fine-tuning can be expected in gauge mediated SUSY breaking models in the light of recent Higgs boson discovery.

We also redo the same calculation in mSUGRA model using updated experimental bound on superpartner masses [22, 23], and check how calculating fine-tuning using stability of Higgs mass rather than usual  $Z$  mass can improve these results.

## 2 Electroweak breaking in MSSM and fine-tuning

Neutral part of the scalar potential in MSSM takes the form

$$V = (\mu^2 + m_{H_u}^2)|H_u|^2 + (\mu^2 + m_{H_d}^2)|H_d|^2 + (bH_uH_d + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)(|H_u|^2 - |H_d|^2)^2. \quad (2.1)$$

Naturalness problem appears in MSSM when we require that the above potential gives correct electroweak symmetry breaking, which gives us  $Z$  boson mass in terms of supersymmetric parameters

$$M_Z^2 = \tan 2\beta (m_{H_u}^2 \tan \beta - m_{H_d}^2 \cot \beta) - 2\mu^2. \quad (2.2)$$

Pushing light Higgs mass to the observed value of 126 GeV requires large radiative corrections, the biggest one comes from top-stop loop [24–26]

$$\delta m_h^2 = \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \left[ \log \left( \frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right], \quad (2.3)$$

where  $M_S^2 = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$  is the average of stop masses, and  $X_t = m_t(A_t - \mu \cot \beta)$  is an off diagonal element of stop mass matrix. Parameters in (2.2) also receive top-stop loop corrections

$$\delta m_{H_u}^2|_{\text{stop}} = -\frac{3Y_t^2}{8\pi^2} (m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2) \log \left( \frac{M_u}{\text{TeV}} \right), \quad (2.4)$$

where  $m_{Q_3}^2$ ,  $m_{U_3}^2$  and  $A_t$  are supersymmetric parameters that predict the stop mass, and  $M_u$  is a scale at which soft masses are generated.

So requiring correct Higgs mass gives large corrections that have to cancel out on the right hand side of (2.2) to give the correct  $M_Z$ .

We define fine-tuning measure with respect to parameter  $a$  as follows<sup>1</sup> [27]

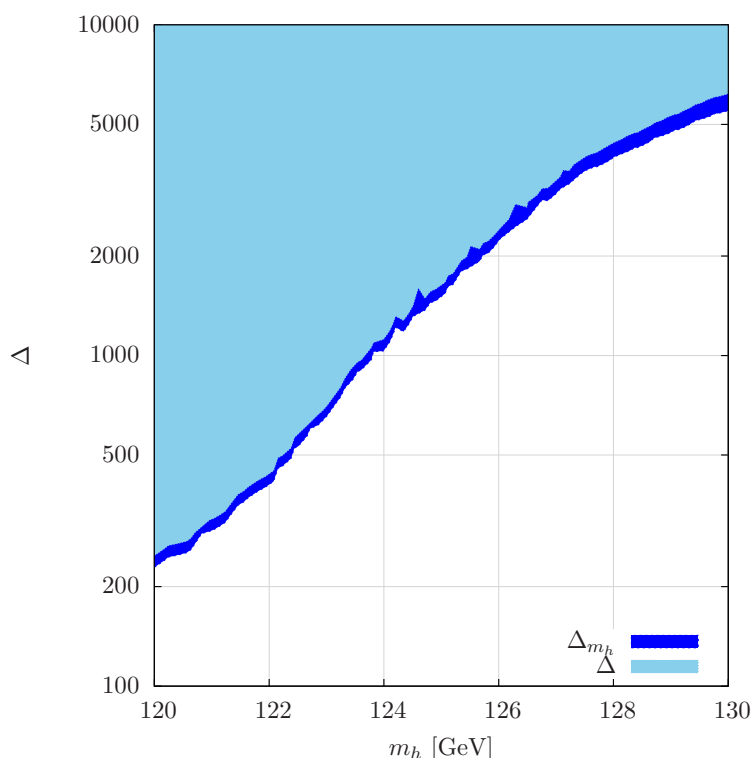
$$\Delta_a = \left| \frac{\partial \ln M_Z^2}{\partial \ln a} \right|. \quad (2.5)$$

Fine-tuning connected with a set of independent parameters  $a_i$  is then<sup>2</sup>

$$\Delta = \max_{a_i} \Delta_{a_i}. \quad (2.6)$$

<sup>1</sup>The numerical procedure used to calculate fine-tuning is detailed in appendix A.6.

<sup>2</sup>Our measure describes sensitivity of electro-weak scale with respect to parameter that would destabilize it most significantly. We did not use a measure that would sum up fine-tuning from all free parameters because we discuss models with at most eight parameters and with at most three of them contributing significantly.



**Figure 1.** Fine-tuning from Higgs mass and  $Z$  mass in mSUGRA model with soft terms generated at scale  $M_u = 2.5 \times 10^{16} \text{ GeV}$  and with  $\tan \beta = 40$ .

Remembering that fine-tuning in the Standard Model actually appeared in the Higgs boson mass we can define fine-tuning with respect to Higgs mass in MSSM

$$\Delta_{h \ a} = \left| \frac{\partial \ln m_h^2}{\partial \ln a} \right| \ ; \ \Delta_h = \max_{a_i} \Delta_{h \ a_i}, \quad (2.7)$$

and calculate it numerically similarly to fine-tuning with respect to  $Z$  mass. In our figures we plot several regions of allowed solutions corresponding to different models on top of each other, because only the borders of these regions (corresponding to the minimal fine-tuning) are actually important. Figure 1 shows that as expected fine-tuning from Higgs boson mass turns out to be similar to the one obtained from  $Z$  boson mass and usually is a few percent lower.

### 3 Gravity vs gauge mediation

Meade, Shih and Seiberg [14] defined gauge mediated models as those in which visible and hidden sectors decouple when gauge couplings vanish. They also have shown that in general such models can only have six parameters determining the low energy sparticle spectrum. In this work we parametrise the high energy soft SUSY breaking terms with three parameters corresponding to gaugino masses

$$M_1 = \frac{\alpha_1}{4\pi} m_Y, \quad M_2 = \frac{\alpha_2}{4\pi} m_w, \quad M_3 = \frac{\alpha_3}{4\pi} m_c, \quad (3.1)$$

and three parameters determining scalar masses  $\Lambda_c^2$ ,  $\Lambda_w^2$ ,  $\Lambda_Y^2$  which give

$$m_f^2 = 2 \left[ C_3(f) \left( \frac{\alpha_3}{4\pi} \right)^2 \Lambda_c^2 + C_2(f) \left( \frac{\alpha_2}{4\pi} \right)^2 \Lambda_w^2 + C_1(f) \left( \frac{\alpha_1}{4\pi} \right)^2 \Lambda_Y^2 \right], \quad (3.2)$$

where  $\alpha_i = g_i^2/4\pi^2$  and

$$\begin{aligned} C_1(f) &= \frac{3}{5} Y_f^2 \\ C_2(f) &= \begin{cases} \frac{3}{4} & \text{for } f = Q, L, H_u, H_d \\ 0 & \text{for } f = U, D, E \end{cases} \\ C_3(f) &= \begin{cases} \frac{4}{3} & \text{for } f = Q, U, D \\ 0 & \text{for } f = E, L, H_u, H_d. \end{cases} \end{aligned} \quad (3.3)$$

Parameters above are assumed do be independent of each other at the high scale so the full set of parameters used in fine-tuning calculation is as follows

$$a_i = \{m_Y, m_w, m_c, \Lambda_Y, \Lambda_w, \Lambda_c, \mu, B_\mu\}. \quad (3.4)$$

A specific model of gauge mediation gives above quantities in terms of physical parameters present in the model. As an example we use two of the models published in [28], the first of which (GGM1) is defined by the superpotential

$$W_1 = X_i (y^i \bar{Q} Q + r^i \bar{U} U + s^i \bar{E} E), \quad (3.5)$$

with three independent parameters used to calculate soft masses

$$\Lambda_Q = \frac{y^i F_i}{y^j X_j} \quad \Lambda_U = \frac{r^i F_i}{r^j X_j} \quad \Lambda_E = \frac{s^i F_i}{s^j X_j}. \quad (3.6)$$

In terms of which soft masses take the form

$$\begin{aligned} m_c &= 2\Lambda_Q + \Lambda_U, & m_w &= 3\Lambda_Q, & m_Y &= \frac{4}{3}\Lambda_Q + \frac{8}{3}\Lambda_U + 2\Lambda_E, \\ \Lambda_c^2 &= 2\Lambda_Q^2 + \Lambda_U^2, & \Lambda_w^2 &= 3\Lambda_Q^2, & \Lambda_Y^2 &= \frac{4}{3}\Lambda_Q^2 + \frac{8}{3}\Lambda_U^2 + 2\Lambda_E^2, \end{aligned} \quad (3.7)$$

and full set of parameters used in fine-tuning calculation is as follows

$$a_i = \{\Lambda_Q, \Lambda_U, \Lambda_E, \mu, B_\mu\}. \quad (3.8)$$

The second model (GGM2) is defined by

$$W_2 = X_i (y^i \bar{Q} Q + r^i \bar{U} U + s^i \bar{E} E + \lambda_q^i q \tilde{q} + \lambda_l^i l \tilde{l}) + F^i X_i, \quad (3.9)$$

with five independent parameters used to calculate soft masses

$$\Lambda_Q = \frac{y^i F_i}{y^j X_j} \quad \Lambda_U = \frac{r^i F_i}{r^j X_j} \quad \Lambda_E = \frac{s^i F_i}{s^j X_j} \quad \Lambda_q = \frac{\lambda_q^i F_i}{\lambda_q^j X_j} \quad \Lambda_l = \frac{\lambda_l^i F_i}{\lambda_l^j X_j}. \quad (3.10)$$

Again we obtain soft masses of the form

$$\begin{aligned} m_c &= \Lambda_q + 2\Lambda_Q + \Lambda_U, & m_w &= \Lambda_l + 3\Lambda_Q, & m_Y &= \frac{2}{3}\Lambda_q + \Lambda_l + \frac{4}{3}\Lambda_Q + \frac{8}{3}\Lambda_U + 2\Lambda_E, \\ \Lambda_c^2 &= \Lambda_q^2 + 2\Lambda_Q^2 + \Lambda_U^2, & \Lambda_w^2 &= \Lambda_l^2 + 3\Lambda_Q^2, & \Lambda_Y^2 &= \frac{2}{3}\Lambda_q^2 + \Lambda_l^2 + \frac{4}{3}\Lambda_Q^2 + \frac{8}{3}\Lambda_U^2 + 2\Lambda_E^2, \end{aligned} \quad (3.11)$$

and full set of parameters used in fine-tuning calculation is as follows

$$a_i = \{\Lambda_Q, \Lambda_U, \Lambda_E, \Lambda_q, \Lambda_l, \mu, B_\mu\}. \quad (3.12)$$

Main disadvantage of gauge mediation in respect of fine-tuning comes from the fact that only negligible  $A$ -terms are generated at SUSY breaking scale. Large mixing in the sfermion mass matrices would increase its contribution to Higgs mass as in eq.(2.3), and make it easier to achieve the experimental result of Higgs boson mass. On the other hand prediction of nonuniversal gaugino masses makes it easier to avoid experimental bound on gluino mass. Nonuniversal scalar masses help avoiding bounds on masses of the first and second generation squarks. We use the following bounds on sparticle masses [22, 23]

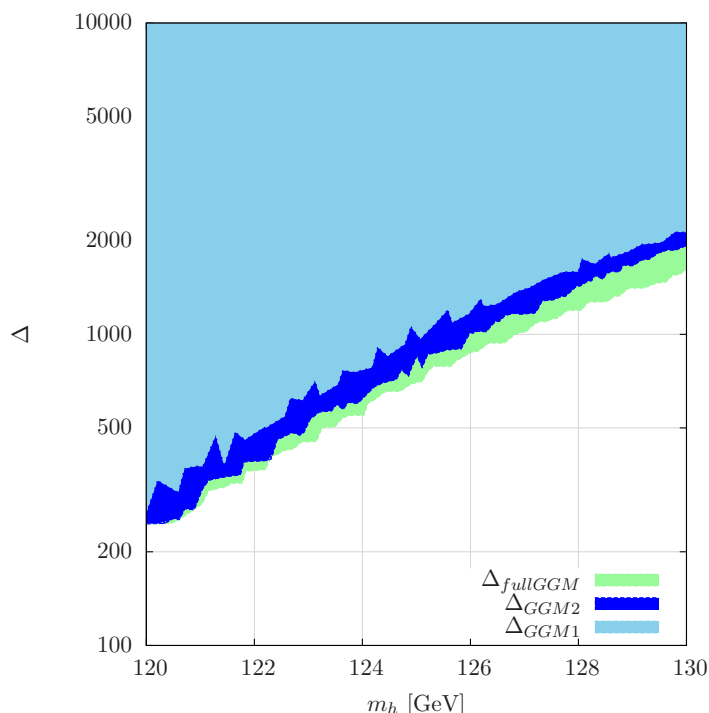
$$\begin{aligned} m_{\tilde{g}} &\geq 1500\text{GeV}, \\ m_{\tilde{u}_i}, m_{\tilde{d}_i}, m_{\tilde{c}_i}, m_{\tilde{s}_i} &\geq 1500\text{GeV} \quad i = 1, 2, \\ m_{\tilde{t}_i} &\geq 560\text{GeV} \quad i = 1, 2, \\ m_{\tilde{b}_i} &\geq 620\text{GeV} \quad i = 1, 2, \\ m_{\tilde{\chi}_1} &\geq 250\text{GeV}. \end{aligned} \quad (3.13)$$

We also assume  $M_u = 10^8\text{GeV}$  and  $\tan\beta = 40$ . Figure 2 shows that generally models with larger number of free parameters predict smaller fine-tuning because they allow to increase Higgs boson mass with subdominant corrections. We also checked that varying the scale  $M_u$  between  $10^6$  and  $10^{12}$  dose not change the shape of that result, while the value of lowest possible fine-tuning changes by up to thirty percent (with lower scales predicting smaller fine-tuning).

In a general model with 6 parameters, the biggest sources of fine-tuning are the gluon mass parameter  $m_c$  or contributions to scalar masses connected with color  $\Lambda_c^2$  or weak interactions  $\Lambda_w^2$ . The  $\mu$  parameter contribution is small in solutions that minimize fine-tuning for a given Higgs mass (which are points that constitute the lower border of allowed fine-tuning regions in our plots), because it actually depends on the value of  $\mu$ . This value can be decreased by increasing  $\Lambda_Y^2$  and  $\Lambda_w^2$  and decreasing  $\Lambda_c^2$  which increases high scale  $m_{H_u}^2$  while keeping masses of coloured particles fixed. The value of  $\mu$  decreases with decreasing energy scale and eventually runs negative to secure correct electro-weak symmetry breaking, as we can see from large  $\tan\beta$  approximation of (2.2)

$$\frac{m_Z^2}{2} \approx -m_{H_u^2} - |\mu|^2. \quad (3.14)$$

As we can see, increasing high scale  $m_{H_u}^2$  makes it run down towards smaller negative value and so decreases  $\mu$  required to obtain correct  $Z$  mass. Since colored particle masses



**Figure 2.** Fine-tuning in models GGM1 and GGM2 as well as in the general six parameter case.

that would increase overall fine-tuning aren't changed, we obtain a scenario with smaller  $\mu$  parameter and similar fine-tuning. Meanwhile, increased  $\Lambda_Y^2$  and  $\Lambda_w^2$  give us larger sub dominant corrections to Higgs mass due to increased masses of non coloured particles. The contribution from  $B_\mu$  parameter is usually small since it enters fine-tuning calculation only through minimization condition of the scalar potential, that gives us new value of  $\tan\beta$  coming from changed supersymmetric parameters. And so the result is suppressed by a factor coming from (2.2)

$$\frac{\partial}{\partial \tan\beta} \tan 2\beta \tan\beta = \frac{\partial}{\partial \tan\beta} \frac{\tan^2\beta}{1 - \tan^2\beta} = \frac{2 \tan\beta}{(1 - \tan^2\beta)^2} \propto \frac{1}{\tan^3\beta} \quad (3.15)$$

which is small for large  $\tan\beta$ .

In model GGM2 squark and gluino masses obtain contributions from all parameters connected with color interactions  $\Lambda_Q, \Lambda_U, \Lambda_q$  and fine-tuning coming from these masses is distributed among these fundamental parameters. The largest fine-tuning contribution turns out to come typically from the  $\mu$  parameter and can come from one of the parameters connected with color only if said parameter is much larger than the other two.

The same can be said about the model GGM1. The biggest source of fine-tuning is usually  $\mu$ , except cases where one of the other parameters is much larger than the other two.

## 4 Reduction of fine-tuning in GGM

The simplest way of reducing fine-tuning is assuming we are considering a model that predicts the parameters which are not independent of each other, but instead are functions

of some fundamental parameters. For example, if gaugino masses  $M_i$  are given functions of parameter  $M_{\frac{1}{2}}$  we obtain

$$\begin{aligned}
 M_i &= f_i(M_{\frac{1}{2}}), \\
 \frac{1}{2}\Delta_M &= \left| \frac{\partial \ln M_Z}{\partial \ln M_{\frac{1}{2}}} \right| = \left| \frac{M_{\frac{1}{2}}}{M_Z} \frac{\partial M_Z}{\partial M_{\frac{1}{2}}} \right| = \left| \frac{M_{\frac{1}{2}}}{M_Z} \frac{\partial M_Z}{\partial M_i} \frac{\partial M_i}{\partial M_{\frac{1}{2}}} \right| \\
 &= \left| \frac{M_{\frac{1}{2}}}{M_Z} \frac{M_i}{M_i} f'_i(M_{\frac{1}{2}}) \frac{\partial M_Z}{\partial M_i} \right| = \left| M_{\frac{1}{2}} \frac{f'_i(M_{\frac{1}{2}})}{f_i(M_{\frac{1}{2}})} \frac{M_i}{M_Z} \frac{\partial M_Z}{\partial M_i} \right| \\
 &= \left| M_{\frac{1}{2}} \frac{f'_i(M_{\frac{1}{2}})}{f_i(M_{\frac{1}{2}})} \frac{\partial \ln M_Z}{\partial \ln M_i} \right| = \left| \sum_{i=1}^3 c_i(M_{\frac{1}{2}}) \frac{\partial \ln M_Z}{\partial \ln M_i} \right|.
 \end{aligned} \tag{4.1}$$

If  $f_i$  are simply proportional to  $M_{\frac{1}{2}}$  one finds

$$\Delta_M = \left| \sum_{i=1}^3 \frac{\partial \ln M_Z^2}{\partial \ln M_i} \right|. \tag{4.2}$$

If these functions were logarithms

$$\begin{aligned}
 f_i(M_{\frac{1}{2}}) &= \tilde{m} \ln \frac{M_{\frac{1}{2}}}{Q}, \\
 c_i(M_{\frac{1}{2}}) &= \frac{\tilde{m}}{M_i}.
 \end{aligned} \tag{4.3}$$

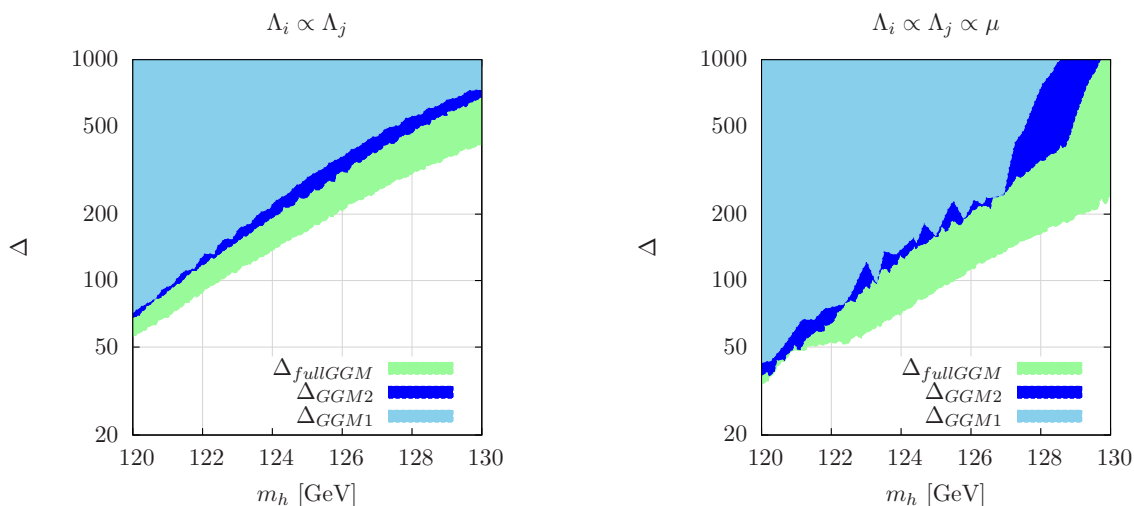
Keeping in mind that fine-tuning is proportional to soft terms  $\Delta_{M_i} \propto M_i$ , we obtain  $\Delta_{M_{\frac{1}{2}}} \propto \tilde{m}$ . However to reduce fine-tuning that way  $\tilde{m}$  would have to be safe from fluctuations. If it was not, we would also have to calculate fine-tuning from  $\tilde{m}$  assuming  $f_i$  from (4.3) are functions of  $\tilde{m}$  which means that logarithms in (4.3) are just proportionality factors and we obtain the same result as for soft terms proportional to each other in (4.2). Keeping that in mind we check only how the usual proportionality of nonuniversal soft masses and  $\mu$  parameter can reduce fine-tuning in models GGM1 and GGM2 as well as in the general case. As one can see from figure 3, simple proportionality of soft terms can greatly decrease fine-tuning in GGM but one still finds  $\Delta > 100$  for  $m_h = 126$ , even in the most general case. We have also checked that in models considered here (for example GGM1 in figure 4) fine tuning coming only from the gauge mediated soft terms can cancel out very precisely if they are proportional to one another, as pointed out in [29].

## 5 Constraints from $g_\mu - 2$

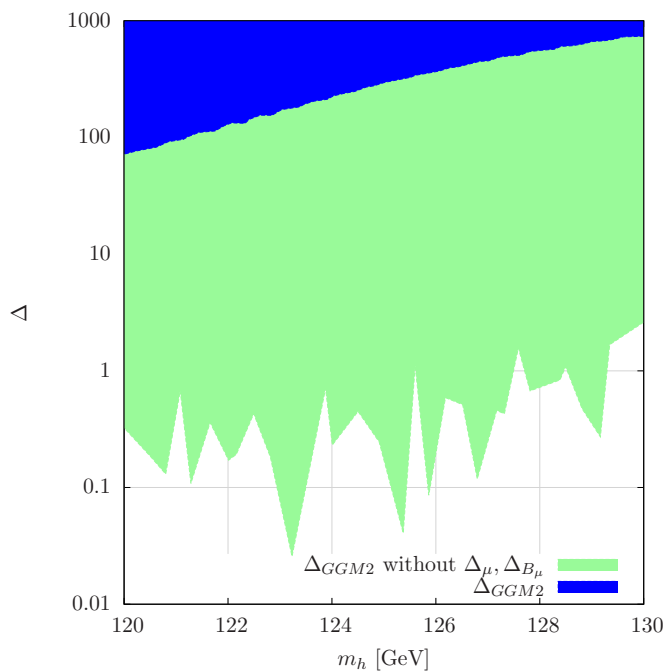
In this section we check whether discussed models can accommodate the discrepancy between measured muon magnetic moment and the standard model prediction [30, 31]

$$\delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (2.8 \pm 0.8) 10^{-9}. \tag{5.1}$$





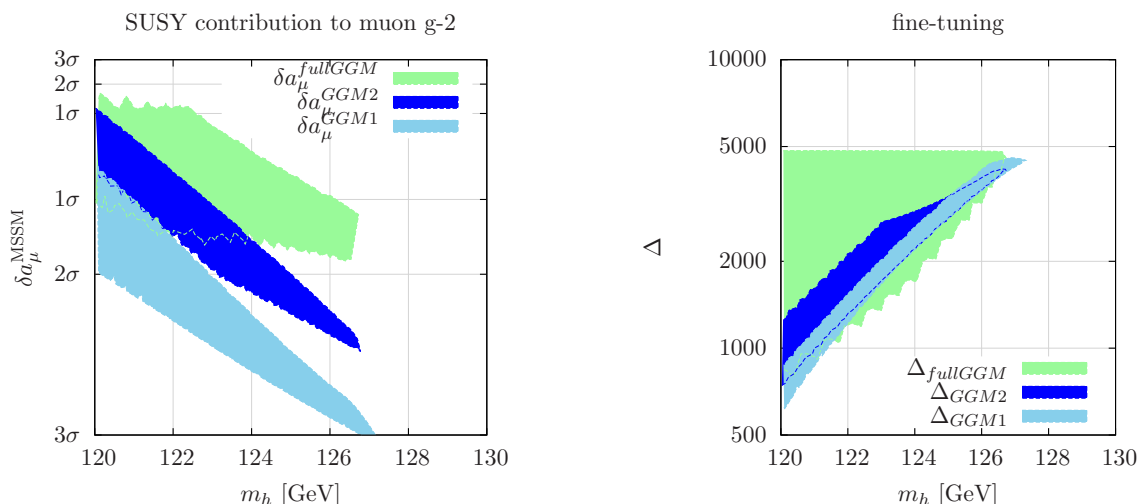
**Figure 3.** Fine-tuning in models GGM1 and GGM2 as well as in the general six parameter case with parameters defining superparticle spectrum proportional to each other and to the  $\mu$  parameter.



**Figure 4.** Fine-tuning in model GGM1 with and without contribution to fine-tuning from  $\mu$  and  $B_\mu$  parameters and with soft terms proportional to each other.

The simplest approximation of supersymmetric contribution to muon magnetic moment is obtained by assuming that  $\tan\beta$  is large and all masses in slepton sector are equal to  $M_{\text{SUSY}}$ . This way one obtains [32]

$$\delta a_\mu^{\text{SUSY}} \approx \left( \frac{g_1^2 - g_2^2}{192\pi^2} + \frac{g_2^2}{32\pi^2} \right) \frac{m_\mu^2}{M_{\text{SUSY}}^2} \tan\beta, \quad (5.2)$$



**Figure 5.** Regions of largest possible SUSY contribution to muon g-2 and corresponding fine-tuning

which indicates a problem since Higgs boson mass depends on soft breaking terms only logarithmically. We evaluate  $\delta a_\mu^{\text{SUSY}}$  numerically using full 1-loop SUSY corrections and 2-loop QED logarithmic corrections from [32]. From figure 2 we can see that only the general case predicts  $\delta a_\mu$  within  $1\sigma$  bound for  $m_h = 126$ , while other models fall out of  $2\sigma$  bounds. Even in the most general case it is hard to increase  $\delta a_\mu$  because all slepton generations have the same mass at the high scale. The 3rd generation gets negative contribution from large Yukawa coupling

$$16\pi^2 \frac{d}{dt} m_{L_3}^2 \supset 2|h_\tau|^2 (m_{H_d}^2 + m_{L_3}^2 + m_{E_3}^2 + A_\tau^2) \quad (5.3)$$

which can make stau tachionic before smuon is light enough to produce the required value of  $\delta a_\mu$ .

Also requiring small masses in slepton sector means we can only increase Higgs mass with dominant squark corrections which increase fine-tuning. And we are left only with solutions with much higher fine-tuning than those that use sub dominant corrections to Higgs mass which we described in previous chapters.

## 6 Summary and conclusions

We have reanalyzed the issue of fine-tuning in supersymmetric models which feature Generalized Gauge Mediation (GGM) in the light of recent discovery of the 126 GeV Higgs particle and taking into account available data on the value of the muon magnetic moment  $g_\mu - 2$ . We consider GGM models with 3, 5 and 6 input parameters and reduce the fine-tuning by assuming simple relations between them at the high scale. We are able to find solutions which give the correct value of the light Higgs mass and are less fine-tuned than models with standard gauge mediation, however one never obtains fine-tung measure

lower than about  $10^2$  if one neglects the data on  $g_\mu - 2$  and about four times more if one takes the constraint given by  $g_\mu - 2$  into account. In general the current  $g_\mu - 2$  data push the models towards high fine-tuning region. However, it is interesting to study the fine-tuning after removing the contributions to the fine-tuning induced by  $\mu$  and  $B_\mu$ , since it isn't obvious that the origin of these two parameters has anything to do with gauge mediation. It is interesting to note, that once this is done, then in the case with neglected  $g_\mu - 2$  constraint one can easily find realistic vacua with purely gauge mediated fine-tuning of order 1 or lower, while the fine-tuning remains always large when the  $g_\mu - 2$  constraint is enforced. One should note, that in the last case even a small shift of the light Higgs mass towards smaller values both reduces fine-tuning and helps to improve agreement of a model with  $g_\mu - 2$  data. Decrease of the Higgs mass down to 123 GeV reduces the fine-tuning by a factor of 2.

To sum up, in models featuring GGM one can naturally obtain fine-tuning smaller than that in models with gravity mediation, despite vanishing A-terms at the high scale. Moreover, considering exclusively fine-tuning coming from gauge-mediated soft masses one can easily achieve arbitrarily small fine-tuning while staying with the correct value of the light Higgs mass. Imposing the agreement of the model with the  $g_\mu - 2$  data restricts parameter space to the region of enlarged fine-tuning, but it is possible to find models which fit into the  $1\sigma$  band. Even a small decrease of the measured value of the Higgs mass would allow for much better agreement of GGM models with measured  $g_\mu - 2$ .

## Acknowledgments

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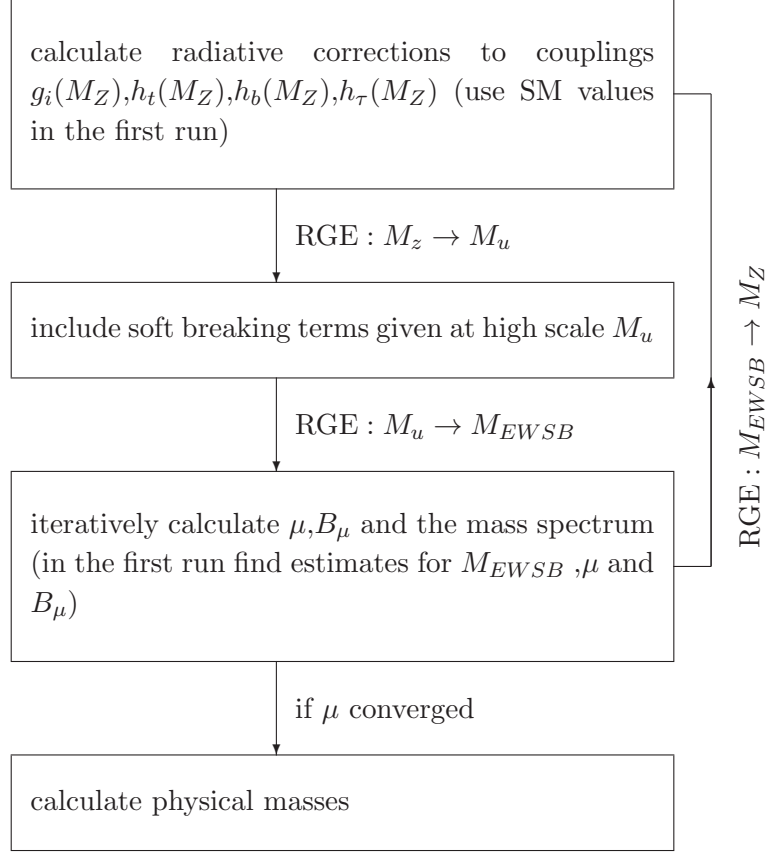
## A Numerical procedure

The numerical procedure we used is similar to the ones used in existing codes like [19–21]. We work with quantities renormalized in  $\overline{DR}$  and use renormalization group equations (RGE), to iteratively find low energy parameters for a given set of high energy set terms.

### A.1 $M_Z$ scale

At  $M_Z$  scale we include radiative corrections to couplings. We set Yukawa couplings using tree level relations

$$h_t = \frac{m_t\sqrt{2}}{v \sin \beta} \quad , \quad h_b = \frac{m_b\sqrt{2}}{v \cos \beta} \quad , \quad h_\tau = \frac{m_\tau\sqrt{2}}{v \cos \beta}, \quad (\text{A.1})$$



**Figure 6.** Schematic of the algorithm we used. all the steps are described in the appendix.

where  $m_t, m_b, m_\tau$  are fermion masses and  $v$  is the Higgs field vev expectation value. At first iteration we use physical masses and SM Higgs vev  $v \approx 246, 22$ . At next iterations above quantities are renormalized in  $\overline{DR}$  scheme and one-loop corrections are included. To calculate top mass we use 2-loop QCD corrections [34] and 1-loop corrections from superpartners from the appendix of [35]. While calculating bottom mass we follow *Les Houches Accord* [36], starting from running mass in  $\overline{MS}$  scheme in SM  $m_b^{\overline{MS}}$ . Next applying procedure described in [37] we find  $\overline{DR}$  mass at  $M_Z$ , from which we get MSSM value by including corrections described in appendix D of [35]. While calculating tau mass we include only leading corrections from [35]. We calculate Higgs vev in MSSM using

$$v^2 = 4 \frac{M_Z^2 + \Re \Pi_{ZZ}^T(M_Z)}{g_2^2 + 3g_1^2/5}, \quad (\text{A.2})$$

where we include  $Z$  self interactions described in appendix D of [35]. To calculate  $g_1, g_2, g_3$  in  $\overline{DR}$  in MSSM we use procedure described in appendix C of [35].

## A.2 RGE and $M_u$ scale

after calculating coupling constants at  $M_Z$  scale we numerically solve renormalization group equations [33, 38], to find their values at  $M_u$  scale, at which we include the soft breaking terms. Then we solve RGEs again to find soft terms, coupling constants,  $\tan\beta$  and Higgs vev  $v$  at scale  $M_{EWSB} = \sqrt{m_{\tilde{t}_1}(M_{EWSB})m_{\tilde{t}_2}(M_{EWSB})}$ . At first iteration we take  $\mu = \text{sgn}(\mu)1\text{GeV}$  and  $B_\mu = 0$  and run to scale at which the above equation is fulfilled.

## A.3 Electro-weak symmetry breaking

In order to obtain correct electro-weak symmetry breaking we use minimization conditions for the scalar potential to find new values of  $\mu$  and  $B_\mu$ . We include radiative corrections in these equations by the substitution

$$m_{H_u} \rightarrow m_{H_u} + \frac{t_u}{v_u} \quad , \quad m_{H_d} \rightarrow m_{H_d} + \frac{t_d}{v_d}. \quad (\text{A.3})$$

We include full one-loop corrections to  $t_u$  and  $t_d$  presented in appendix E of [35] and leading two-loop corrections [39–43]. Since these corrections depend on sparticle masses which in turn depend on  $\mu$  parameter that we aim to calculate, an iterative calculation is performed to obtain new values of  $\mu$  and  $B_\mu$ .

If the new values differ significantly from the ones obtained in previous repetition of the whole algorithm described above, we run back to the  $M_Z$  scale and repeat the whole calculation once again. If however the values of  $\mu$  and  $B_\mu$  converged, we can move on to calculation of physical masses.

## A.4 Calculation of physical masses

To calculate physical masses we use only leading corrections described in [35] everywhere but the Higgs sector. In the Higgs masses calculation we use full one-loop corrections from [35] and leading two-loop corrections described in [39–43].

## A.5 Constraints imposed on the scalar potential

To check if a given set of soft terms describes a realistic physical situation we check if the scalar potential is not unbounded from below (UFB). And if the potential does not have minimums deeper than the one breaking electro-weak symmetry, which would break  $\text{SU}(3)$  or  $\text{U}(1)_{\text{em}}$  (CCB). [44–48]. We include simple tree level bounds:

- for UFB

$$|\mu B_\mu| \leq m_{H_u}^2 + m_{H_d}^2 \quad \text{at scale} \quad M_x \in [M_{EWSB}, M_u], \quad (\text{A.4})$$

- and CCB

$$A_f^2 \leq 3(m_{f_L}^2 + m_{f_R}^2 + \mu^2 + m_{H_u}^2) \quad \text{at scale} \quad M_x \in [M_{EWSB}, M_u]. \quad (\text{A.5})$$

## A.6 Fine-tuning

After the calculation of the spectrum is finished, one has a whole set of parameters and couplings that predict correct electro-weak symmetry breaking. In order to calculate fine-tuning we solve RGE from  $M_u$  scale down to  $M_{EWSB}$  with one of the fundamental parameters  $a_i$  changed slightly at the high scale  $M_u$ . Then at the scale  $M_{EWSB}$  we recalculate the spectrum and use minimization conditions to calculate new value of  $\tan\beta$  and to obtain our new prediction for  $m_Z^2$ , which means that we calculate numerically the derivative in the definition of fine-tuning (2.5). We repeat that procedure for all parameters  $a_i$  and obtain our final result as a maximum of results obtained for each of those parameters (as in (2.6)).

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